



Multi-objective Optimization of Contingency Logistics Networks with Distorted Risks

Esra Dağ¹, Mehmet Miman²

Abstract

This study develops a mathematical formulation of multi-objective optimization of contingency logistics networks (CLN) with stock allocation, cost of allocation and network reliability using weighted objective function and applying distorted risk. The model first time analyze such systems and provide illustrative case for the solution of such systems through Excel Solver with Non-linear optimization option as the model provided in this study is a non-linear, non-convex, and non-separable. The solution found for illustrative case is an Pareto optimum depending on the weights selected.

Keywords: Multi-objective, Optimization, CLN

Introduction

Thomas, [1], appears to be the first person who explicitly considers the reliability of a contingency logistic system where he defines a contingency as an “unexpected crises event that creates a major threat to the safety and security of a population”. As he indicates, they range from military conflicts that require engagement with hostile forces, police actions for civil disorder, to humanitarian relief of victims from disasters such as earthquakes, hurricanes, floods, and related catastrophes. The reliability of base is defined based on interference theory between supply and demand carried to and emerged from the base. In a contingency setting demand emerged from operational base and supply that can be carried to the base are random and unknown beforehand. If the supply to the base is less than demand emerged the base is called as “failed” as the base is incapable of performing the associated contingency operation. Miman and Pohl, [2], extend Thomas’s [1] work by including stock allocation in Contingency Logistics Network (CLN) in planning stage and provide the formula for the reliability of the base which are allowed to hold stocks based on inference theory.

Offut et. al, [3], provide distorted risks models namely proportional hazard (PH) and Dual Power (DP) models. Miman and Pohl, [2], provide the distorted risks analysis according to risk averseness of the decision makers for CLNs and provide multi-dimensional knapsack approach for natural vulnerability of the system defined in their study.

In the literature, Miman and Pohl, [4], provide a multi-objective optimization of CLNs through physical programming in the context of selective maintenance. For the same context, Dağ and Miman, [5], provide the multi-objective optimization modeling of CLNs through utopia-distance.

This study will provide the multi-objective formulation of the system and its solution through Excel Solver. First a Notation list is provided, then mathematical formulation is developed. Based on this, an optimization model is constructed, and its solution is illustrated using the same case as Miman and Pohl, [2], used but formulating the problem as a multi-objective optimization model.

¹ Esra Dağ, Toros University, Faculty of Engineering, Department of Industrial Engineering, Mersin, Turkey,
g.esra.dag@toros.edu.tr

² Mehmet Miman, Toros University, Faculty of Engineering, Department of Industrial Engineering, Mersin, Turkey,
mehmet.miman@toros.edu.tr



Model Development

This section provides all the notation list and mathematical formulation necessary to model the multi-objective optimization in CLN with distorted risk.

Notation List

S_i	Random variable representing the supply at site $i = 1, \dots, n$
D_i	Random variable representing the demand at site $i = 1, \dots, n$
X_i	Probability distribution of the supply at site $i = 1, \dots, n$
Y_i	Probability distribution of the demand at site $i = 1, \dots, n$
f_i	Density (or probability mass) function of the supply at site $i = 1, \dots, n$
g_i	Density (or probability mass) function of the demand at site $i = 1, \dots, n$
G_i	CDF for the demand at site $i = 1, \dots, n$
μ_i	Exponential rate of the supply for Y_i
λ_i	Exponential rate of the demand for X_i
MD_i	Mean demand at site i ; $1/\lambda_i$
MS_i	Mean supply for site i ; $1/\mu_i$
ψ_i	The probability distribution of excess demand for site $i = 1, \dots, n$
s_i	Safety stock kept at the site $i = 1, \dots, n$
s	Max total stock to be allocated through n sites
c_i^s	Cost of keeping one unit stock at supply site $i = 1, \dots, n$
ρ_i	Failure probability of site $i = 1, \dots, n$
ν_i	Natural vulnerability of site $i = 1, \dots, n$
$\hat{\rho}_i^d$	Distorted risk for site $i = 1, \dots, n$ under the distortion d
R_i	Site reliability, $i = 1, \dots, n$
a_i	Distortion parameter for node i under PH model
b_i	Distortion parameter for node i under DP model
R	Reliability of the CLN
C	Cost of stock allocation in CLN
S	Total number of stocks to be allocated.
α	Weight for reliability
β	Weight for cost
δ	Weight for stock
$\zeta(X)$	Aggregation Function of X

Mathematical Formulation

According to Miman and Pohl, [2], the definition of the failure probability of operational site i can be computed as:

$$\rho_i = \Pr\{D_i - S_i \geq s_i\} = \Pr\{E_i \geq s_i\} = 1 - \psi_i(s_i) \quad (1)$$

ψ_i can be derived by conditioning on S_i . (Assuming exponential supply and demand)



$$\begin{aligned}\psi_i(s_i) &= \Pr\{D_i < S_i + s_i\} = \int_0^\infty Y_i(x + s_i) f_i(x) dx = \int_0^\infty (1 - e^{-\lambda_i(s_i+x)}) \mu_i e^{-\mu_i x} dx \\ &= \int_0^\infty \mu_i e^{-\mu_i x} dx - \mu_i \int_0^\infty e^{-\lambda_i(s_i+x)} e^{-\mu_i x} dx = 1 - \mu_i e^{-\lambda_i s_i} \int_0^\infty e^{-x(\mu_i + \lambda_i)} dx = 1 - \frac{\mu_i}{\mu_i + \lambda_i} e^{-\lambda_i s_i}\end{aligned}\quad (2)$$

Based on Equation (1) and Equation (2), the following risk measure for site i is given by Equation (3).

$$\rho_i = 1 - \psi_i(s_i) = \frac{\mu_i}{\mu_i + \lambda_i} e^{-\lambda_i s_i} \quad (3)$$

The distorted risks can be computed through Equation (4).

$$\begin{aligned}\hat{\rho}_i^{PH} &= (\rho_i)^a, \quad 0 \leq a \leq 1 \\ \hat{\rho}_i^{DP} &= 1 - (1 - \rho_i)^b, \quad b \geq 1\end{aligned}\quad (4)$$

The reliability of the CLN, R , can be derived according to structure function, the cost of stock allocation, C , would be given by Equation (5), while total stocks allocated, S , would be given by Equation (6).

$$C = \sum_i c_i^s s_i \quad (5)$$

$$S = \sum_i s_i \quad (6)$$

In this paradigm, while reliability of network is desired to be maximized, number of stocks allocated and cost of allocations are desired to be minimized. That is there is a trade of between R and S/C . To include all of the criteria in a single objective aggregation function is used for each criteria as provided by Equation (7)

$$\zeta(R) = R, \quad \zeta(S) = S^{-1}, \quad \text{and} \quad \zeta(C) = C^{-1} \quad (7)$$

The resulting weighted objective optimization model can be expressed by P .

$$\begin{aligned}Max \quad & \alpha R + \frac{\beta}{C} + \frac{\delta}{S} \\ s.t \quad & \alpha + \beta + \delta = 1 \\ & s_i \geq 0 \quad \text{integer}, \forall i\end{aligned}$$

Figure 1. Mathematical Model: P

Note that P defines a non-linear, non-convex, and non-separable model.

Illustration

Same example as one used by Miman and Pohl, [2], is used to illustrate the model and its solution. They modeled and solved the CLN as multi-dimensional knapsack problem. Here it is modeled as multi-objective optimization problem.

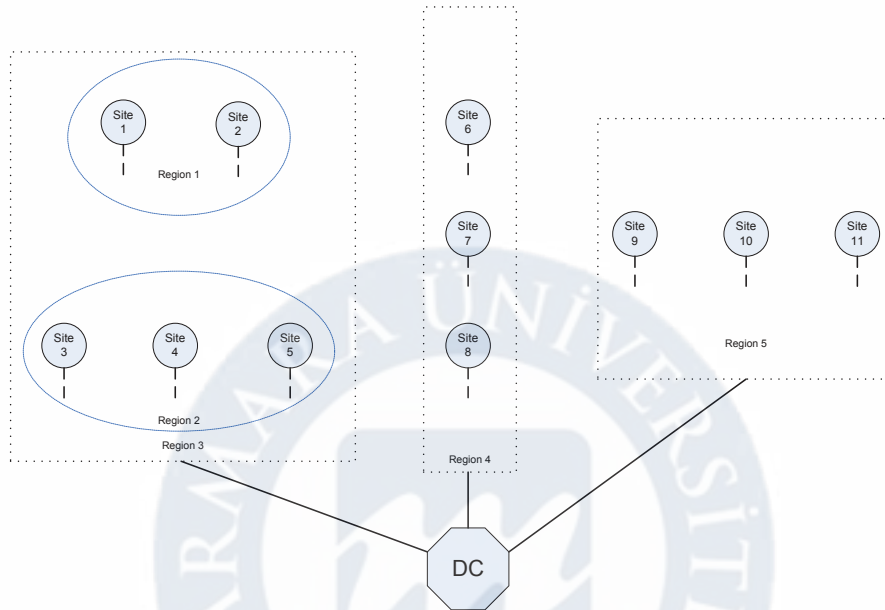


Figure 2. Illustrative Contingency Logistics Network ([2])

In this scenario, the success of the contingency operation is based upon the accomplishment of the sub missions for Regions 3, 4, and 5. These regions consist of a total of 11 operational sites. The accomplishment of each task at each site can be achieved when the site is mission capable, i.e. it has enough items to satisfy the demand for the specific contingency operation.

Region 3 is mission capable when either Region 1 or Region 2 is mission capable. Region 1 is mission capable if both sites one and two are working, similarly Region 2 is mission capable if sites 3, 4, and 5 are all mission capable. Region 4 is mission capable when at least 2 of sites 6, 7, or 8 are mission capable. Finally, Region 5 requires that sites 9, 10, and 11 be capable of performing assigned tasks that require resources supplied from the DC. Demand and supply for the items required to accomplish the assigned tasks, at each operational site i ; $i = 1, 2, \dots, 11$; are assumed to be exponentially distributed with rates λ_i and μ_i , respectively. Table 1 provides the probabilities of being mission capable for each region as well as entire supply chain system where ρ_i is specified by Equation (3).

Table 1. Mission Capability Probabilities for the CLN

Region	Probability of Mission Capability	
1	R_1	$(1-\rho_1)(1-\rho_2)$
2	R_2	$(1-\rho_3)(1-\rho_4)(1-\rho_5)$
3	R_3	$1-(1-R_1)(1-R_2)$
4	R_4	$1-\rho_6\rho_7-\rho_6\rho_8-\rho_7\rho_8-\rho_6\rho_7\rho_8$
5	R_5	$(1-\rho_9)(1-\rho_{10})(1-\rho_{11})$
System	R	$R_3R_4R_5$

For the distorted risk analysis, a PH distortion is used for sites 3, 4, 5, and 7 where the perceived risk is greater for site 7 than for sites 3-5. Therefore, using the PH distortion model, the parameter for site 7 is set at 0.3, which is significantly smaller than the 0.7 used for sites 3-5. The choice of distortion level should reflect the decision maker's risk tolerance for that region. (For the other sites, the decision maker is assumed to be risk neutral, i.e. $\alpha = 1$). The input parameters are given in Table 2. Note that the weight for each of three criteria is set to be 1/3.



Table 2. Inputs for Illustrative Case

	Region	Site	MD	MS	λ	μ	s	c	a
Region 3	1	1	100	80	0.01	0.0125		1,00	1,00
		2	100	125	0.01	0.008		1,50	1,00
	2	3	125	125	0.008	0.008		1,20	0,70
		4	100	125	0.01	0.008		1,20	0,70
		5	150	150	0.00667	0.0067		0.8	0,70
Region 4	4	6	125	150	0.008	0.0067		1,20	1,00
		7	150	150	0.00667	0.0067		1,50	0,30
		8	100	150	0.01	0.0067		1,00	1,00
Region 5	5	9	100	100	0.01	0.01		1,00	1,00
		10	100	125	0.01	0.008		1,10	1,00
		11	125	125	0.008	0.008		1,10	1,00
							Total		

Using Excel Solver, the optimal values are obtained as shown in Figure 3.

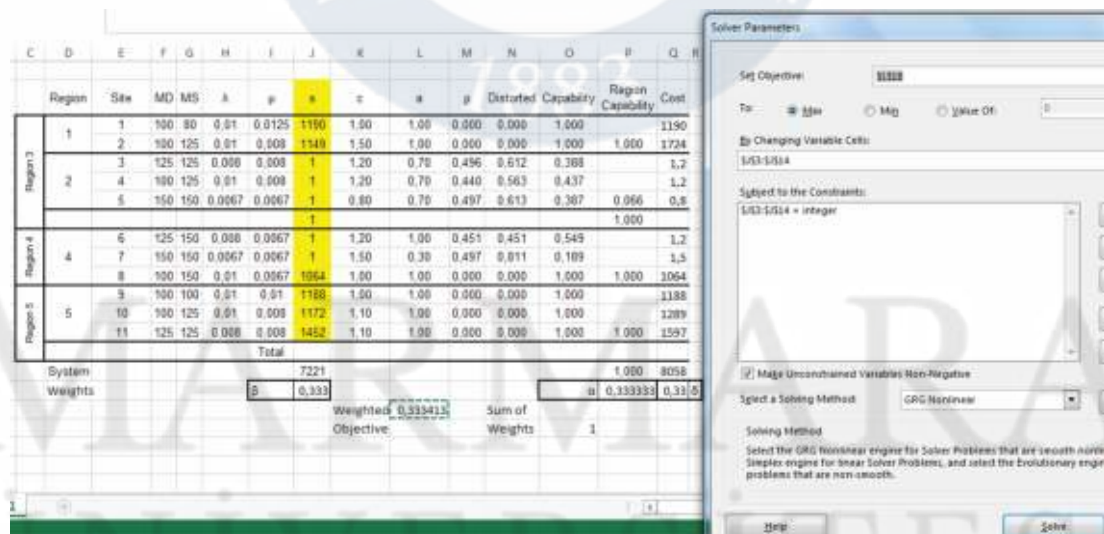


Figure 3. Solution of Illustrative Case through Excel Solver

The optimal solution in the illustrative case enforce reliability to be approximately equal to 1 with a cost of 8058 dollars and with total stocks of 7221 allocated. Note that the solution found is a Pareto optimum.

Conclusion and Discussion

In this study, a multi-objective optimization model with distorted risk in a CLN is provided and illustrated with an example. This study can be regarded as a preliminary study in the field and furthermore extended by including analysis in the selection of weights and aggregation function for each criteria and selection of the values as distortion parameters.



References

- [1] Thomas, M.U., 2004, "Assessing Reliability of Contingency Logistic network", *Military Operation Research*, Vol. 9, No. 1, pp. 33-41.
- [2] Miman, M. and Pohl, E. A., 2008, "Modeling and Analysis of Risk and Reliability for a Contingency Logistics Supply Chain", *Journal of Risk and Reliability*, v222, n4, 477- 494.
- [3] Offut, M.E., Kharoufeh, J. P. and Deckro, R. F., 2006, "Distorted Risk Measures with Application to Military Capability Shortfalls", *Military Operation Research*, Vol. 11, No. 4, pp. 25-39.
- [4] Miman, M. and Pohl, E.A., 2012, "Multi-objective optimisation of a contingency logistics network through physical programming", *International Journal of Collaborative Enterprise*, v3, n1,1-17.
- [5] Dağ, E. ve Miman, M., 2014, "Beklenmedik Durumlar Lojistiğinin Optimizasyonunda Ütopya Uzaklık Metodu", *III. Ulusal Lojistik ve Tedarik Zinciri Kongresi*, pp680-688, 15-17 Mayıs 2014, Trabzon.



MARMARA
ÜNİVERSİTESİ