



An Analysis of Statistical Control Charts with Fuzzy Set Theory

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Abstract

Process control, product control and quality improvement are crucial for manufacturing companies to survive in competitive environment. The aim of quality control is to provide that the products, services, or processes supplied meet customer specifications. In the manufacturing process, it is not always possible to satisfy the customer specifications due to problems related variations in process, materials, machines, etc. Therefore monitoring a process to ensure control is required to manufacture products that close to target value and with little variation and in desired amount is very important. In this study, quality characteristics of items packaging process were investigated using statistical quality control and fuzzy set theory. For this purpose, product samples were taken and classified into four categories as the quality grades such as “Bad”, “Medium”, “Good”, and “Excellent”. Fuzzy logic and probability theory approaches were used for constructing control charts for quality assurance. With the help of these theories, flexibility of the system was improved. Consequently, the based on fuzzy set theory results are discussed with the existing approach to see the difference in performance.

Keywords: Quality Control Charts, Probability Theory, Fuzzy Theory

Introduction

Manufacturing firms are being under pressure on the basis of quality in the competition in the world for the last few years. Therefore, manufacturing firms make an effort to use quality improvement programs to survive in this competitive environment. Statistical process control basically SPC is used as a method that is applied by monitoring and controlling the processes in the quality control. The process is analyzes if it works in its full potential by monitoring and controlling the process. SPC aims to produce conforming product with a minimum rework. Observation, evaluation, diagnosis, decision and implementation are the stages of SPC. Marcucci (1985) proposed two procedures, first one detects the differences in any of the quality proportions by the help of Pearson X^2 , on the other hand, the last one uses the multinomial distribution. In the consideration of fuzzy theory Raz and Wang (1988, 1990a, b) applied fuzzy sets by converting them to the linguistic term. After that, each sample is associated by the use of fuzzy rules. After that process, Shewhart control charts are used to plot the fuzzy set.

Quality control includes to check all steps in a process. Finding defects, testing the output which can be product or service, improving the process, minimizing the defects take first place in the quality control processes to meet specific requirements to be in a satisfactory level. [2] Control charts are the main tools used in the SPC which are generated by Shewhart in 1924. The control charts have upper and lower specification limits where the data is plotted and need to be between the specification limits. In control charts there are an upper control limit, a lower control limit, and a center line that are shown basically as UCL, LCL, CL respectively. Also, three sigma (3σ) control limits are used. When the process plotted above and below the mean by three times of process's standard deviation. When the process is in control or not, control charts are very useful to detect this nonconforming conditions by monitoring the process. If the data is plotted with little variation around the target value, and if it is between the specification limits, then the process is “in control”.

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On the other hand, if the data is plotted with a pattern or if it exceeds the specification limits, then the process is “out of control” which need to take a preventive action to decrease the variability.

In 1999, fuzzy logic is considered as an approximate value which can be achieved instead of a fixed value. That is fuzzy logic variables can take a real value between 0 and 1. [4] In 2014, Ahlawat, N., Ashu G., and Nidhi S. represent a study which is known as the fuzzjjective. They mentioned that when linguistic variables are used the degrees of specific functions can be controlled [3].

In other respects, Zadeh and Klaua explained the fuzzy sets as elements have degrees of membership then the fuzzy sets occur. [5-6] Fuzzy set theory allows the incremental consideration of the elements’ membership in a set that is valued in the real unit interval [0, 1] with an aid of a membership function. When information is not completed or imprecise, the fuzzy set theory is very useful with a numerous domains.

In this study, quality characteristics of items packaging process were investigated using statistical quality control and fuzzy set theory. For this purpose, product samples were taken and classified into four categories such as “Bad”, “Medium”, “Good”, and “Excellent” as quality grades. Fuzzy logic and probability theory approaches were used for constructing control charts for quality assurance. With the help of these theories, flexibility of the system was improved. Consequently, the based on fuzzy set theory results are bench marked to see the diversity between the current situation and the proposed one.

The rest of the paper is organized as follows: Methodology of the study is detailed in next section. After representing the methodology section of the study, there is an application and numerical results of the application. Finally the paper concludes with a summary and outlines scope of research.

In addition, an overview of scientific literature on fuzzy logic and probability theory approaches for control charts are represented in the Table 1.

Table 1. Several Studies in the Literature by Considering the Fuzzy Control Charts

Reference/Year	Name of the Study	Method
[10] / (2013)	“Fuzzy Approach to Statistical Control Charts”	Attribute quality characteristics are monitored.
[11] / (2013)	“Basic Developments of Quality Characteristics Monitoring”	Bias of quality characteristics are monitored
[12] / (2007)	“An alternative approach to fuzzy control charts: direct fuzzy approach”	A proper α - cut is applied as the difficulty of the inspection.
[13] / (2006)	“Development of fuzzy process control charts and fuzzy unnatural pattern analyses”	Direct fuzzy approach (DFA) is used with none transformation method.
[14] / (2006)	“ α -cut fuzzy control charts for linguistic data”	A proper α -level is applied as the difficulty of the inspection.
[15] / (2010)	“Comparing Fuzzy Charts with Probability Charts and Using Them in a Textile Company”	The quality is monitored in terms of conformance and nonconformance in a textile company.
[16] / (2009)	“Construction of quality control charts by using probability and fuzzy approaches and an application in a textile company”	The quality is monitored in terms of conformance in a textile company.

Methodology

In this study, quality characteristics of items packaging process were investigated using statistical quality control and fuzzy set theory. For this purpose, product samples were taken and classified into four categories such as “Bad”, “Medium”, “Good”, and “Excellent” as quality grades. Fuzzy logic and probability theory approaches were used for constructing control charts for quality assurance. With the help of these theories, flexibility of the system was improved. Consequently, the based on fuzzy set theory results are bench marked to see the diversity between the current situation and the proposed one.



While monitoring multinomial processes, there are several procedures proposed as mutually exclusive categories for classified products. One of the useful proposed procedures is Marcucci's procedure that use Shewart type control charts. This type of control chart has separated into two types. The first one is used when specific values are developed for quality proportions. In this first type procedure, with the monitoring process the change can be detected. The second type procedure is designed to detect quality proportions when there is an increase. However, goodness-of-fit statistic of the Pearson cannot be possible to apply when the process proportions are not given. Therefore, proportions' homogeneity among the beginning period, 0 and for every observing period, as shown in Equation (1).

$$Z_i^2 = \sum_{k=i,0} \sum_{j=1}^1 \frac{n_k \left(\frac{X_{kj}}{n_k} - \frac{X_{ij} + X_{0j}}{n_i + n_0} \right)}{\frac{X_{ij} + X_{0j}}{n_i + n_0}} = n_i n_0 \sum_{j=1}^1 \frac{(p_{ij} - p_{0j})^2}{X_{ij} + X_{0j}} \quad (1)$$

where the sample proportions are $k = \{0, 1\}$, $p_{kj} = \frac{X_{kj}}{n_k}$, $j = 1, 2, 3, \dots, t$ and the sample size is n_i .

When fuzzy theory is considered, in 1990 a study is suggested where fuzzy sets are converted to linguistic term with the single fuzzy set result. With the help of Shewart-type control chart, the fuzzy set is generated and the measure centrality of the fuzzy set can be seen easily. Scalar values are generated by the transformation of the fuzzy sets which are assigned to the linguistic values. This is an important point, instead of using the standard format of control charts. [7]. There are a couple of way to see the variety of the in the fuzzy set that includes the base variable. The following four ways is represented as in descriptive statistics.

Fuzzy average, f_{avg} : gives the fuzzy average according to Zadeh [7].

$$f_{avg} = Av(x : F) = \frac{\int_{x=0}^1 x \mu_F(x) dx}{\int_{x=0}^1 x \mu_F(x) dx} \quad (2)$$

Fuzzy mode, f_{mode} : represents the base variable value where F is equal to 1 as the membership function.

$$f_{mod} = \{x \mid \mu_F(x) = 1\}, \forall x \in F \quad (3)$$

Fuzzy median, f_{med} : shows the membership function parts under the fuzzy set. The following formula gives the results of the equal regions where the end points in the base variable of the fuzzy set F is defined as a and b , as $a < b$.

$$\int_a^{f_{med}} \mu_F(x) dx = \int_{f_{med}}^b \mu_F(x) dx = \frac{1}{2} \int_a^b \mu_F(x) dx \quad (4)$$



α -Level fuzzy midrange, $f_{mr}(\alpha)$: A_α is the center of the α level cut where α level cut is a non-fuzzy set. Here the membership of each element is greater than or equal to α . When b_α and a_α are the final points of α level cut, A_α then the following expression is given to determine $f_{mr}(\alpha)$.

$$f_{mr}(\alpha) = \frac{1}{2}(a_\alpha + b_\alpha) \quad (5)$$

There are two approaches which are proposed by Wang and Raz, as every fuzzy subset value is transformed in the member values. The approaches are the fuzzy probabilistic and the membership approach.

For *the membership approach* the following equations are considered.

Equation (7) calculates the sample mean for the j^{th} sample is where average sample mean is the $\overline{M}_j$, and initial number of sample is the m . Also, the number of products as linguistic term L with j sample is defined as k_{ij} ; the fuzzy value as the linguistic term i is represented as r_i and the size of sample j is shown as n_j .

$$M_j = \frac{\sum_{i=1}^m k_{ij} r_i}{n_j}, i = 1, 2, 3, \dots, t \quad (6)$$

The centerline (CL) for sample m of size n is given as follows:

$$CL = \overline{M}_j = \frac{\sum_{j=1}^m M_j}{m} \quad (7)$$

Equation (8) represents the control limits for the below and above the central line as each sample will be at a range between 0, and 1.

$$\begin{aligned} \text{MembershipLCL} &= \text{Max}\{0, [CL - k\delta(GMF)]\} \\ \text{MembershipUCL} &= \text{Min}\{0, [CL + k\delta(GMF)]\} \end{aligned} \quad (8)$$

Also, α is the value of membership and the mean deviation for a given fuzzy set A is $(\delta(A))$. Equation (9) represents the sum of the right mean deviation (δ_r) , and the right mean deviation (δ_r) .

$$\begin{aligned} \delta_l &= \int_{\alpha=0}^1 [x_m - x_l(\alpha)] d\alpha \quad \text{And} \quad \delta_r = \int_{\alpha=0}^1 [x_r - x_m(\alpha)] d\alpha \\ \delta(A) &= \delta_l(A) = \delta_r(A) = \int_{\alpha=0}^1 [x_r - x_m(\alpha)] d\alpha \end{aligned} \quad (9)$$



For the *fuzzy probabilistic approach* the following equations are considered.

Equation (10) shows the sample mean M_j , for each sample j . Moreover, F_i which is the fuzzy subset which shows the r_i value that is transformed from the linguistic terms L_i .

$$M_j = \sum_{i=1}^t r_i k_{ij} \quad (10)$$

The mean of the representative of the m samples, MSD is given as follows.

$$MSD = \frac{1}{m} \sum_{j=1}^m SD_j \quad (11)$$

The standard deviation SD_j is determined for each sample j in Equation (12).

$$SD_j = \sqrt{\frac{1}{n-1} \sum_{i=1}^t k_{ij} (r_i - M_j)^2} \quad (12)$$

The centerline is represented in Equation (13) as the grand mean of the sample means M_j .

$$CL = \frac{\sum_{j=1}^m M_j}{m} = \frac{\sum_{j=1}^m \sum_{i=1}^t r_i k_{ij}}{mn} \quad (13)$$

As the data is generated with sample means on the charts which are member values, the data must be distributed between 0, and 1. Thus, there are two assumptions for control limits. One of them is the sample distribution is normal, and the second one is when the sample size n is large enough such as, it is greater than 25, control limits are represent in the Equation (14) [8].

$$\begin{aligned} LCL &= \text{Max}\{0, (CL - A_3 MSD)\} \\ UCL &= \text{Min}\{0, (CL + A_3 MSD)\} \end{aligned} \quad (14)$$

Comparison Study

In frozen food production, packaging of products is one of the important quality feature that has to be checked continuously [18]. Therefore, for control of the quality, product packaging process was analyzed with 30 samples of size 50. Items packaging are classified by experts using four linguistic terms: Excellent (E), Good (G), Medium (M), and Bad (B). *Generalized p chart* proposed by Marcucci [19], *Probabilistic and Memberships approaches* proposed by Wang and Raz [7], [8] were evaluated in this comparison study. Table 2 shows dataset for 30 samples [20].



Table 2. Dataset [20]

Sample No	Bad	Medium	Good	Excellent	Sample No	Bad	Medium	Good	Excellent
1	8	9	25	8	16	5	9	28	8
2	10	8	25	7	17	7	9	27	7
3	6	8	28	8	18	3	9	30	8
4	6	8	28	8	19	9	9	26	6
5	3	9	28	10	20	7	8	27	8
6	5	9	27	9	21	12	15	20	3
7	10	8	24	8	22	11	11	22	6
8	6	9	27	8	23	17	9	20	4
9	9	9	26	6	24	10	9	24	7
10	7	8	27	8	25	6	9	28	7
11	3	9	30	8	26	7	10	25	8
12	4	10	29	7	27	5	12	25	8
13	11	9	24	6	28	9	8	26	7
14	7	10	26	7	29	5	12	25	8
15	16	9	20	5	30	4	9	28	9

Marcucci Approach

Since the proportions are unknown before; the data conforms to second type of Marcucci case where the Pearson goodness of fit test is not appropriate. Therefore, we supposed that the process is in control in the period 4, and calculated sample proportions of the base period ($p_{01}=0.12$, $p_{02}=0.16$, $p_{03}=0.56$, $p_{04}=0.16$). Then, generalized p chart was tested between base period and other periods of the process. The upper control limit was taken as 95th percentile of the $X^2(3)$ distribution which is equal to 7,815. The generalized p -chart for the 30 samples was illustrated on Figure 1.

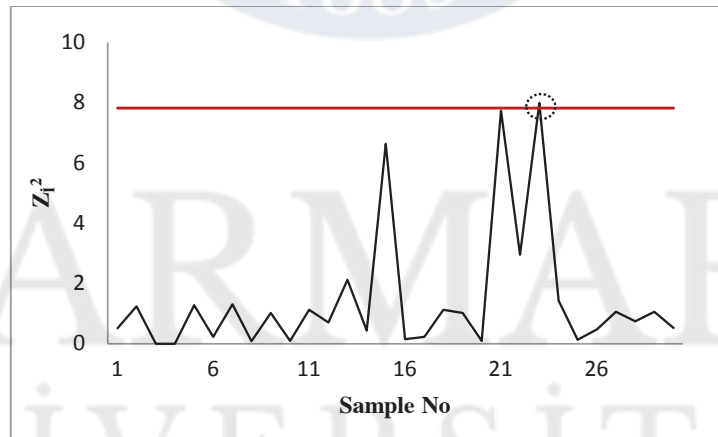


Figure 1. The Generalized P-Chart for the 30 Samples

As seen Figure 1, the process is out of control only one occasion: Samples 23.

Wang and Raz Approaches

In this study, the approaches suggested by Raz and Wang [7], [8] were applied to the dataset. Bad (B), Medium (M), Good (G), and Excellent (E) quality characteristics were defined as the term set. Membership functions was used for each term as shown in Table 3 [17]

Table 3. Membership Functions [17]

$$M_B(x) = \begin{cases} 0, & x \leq 0 \\ -x + 1, & 0 \leq x \leq 1 \\ 0, & x \geq 1 \end{cases} \quad M_M(x) = \begin{cases} 0, & x \leq 0 \\ 4x, & 0 \leq x \leq \frac{1}{4} \\ -\frac{4}{3}x + \frac{4}{3}, & \frac{1}{4} \leq x \leq 1 \\ 0, & x \geq 1 \end{cases}$$

$$M_G(x) = \begin{cases} 0, & x \leq 0 \\ 2x, & 0 \leq x \leq \frac{1}{2} \\ 2 - 2x, & \frac{1}{2} \leq x \leq 1 \\ 0, & x \geq 1 \end{cases} \quad M_E(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 \leq x \leq 1 \\ 0, & x \geq 1 \end{cases}$$

The triangular-shaped membership functions of each term were also constructed on Figure 2.

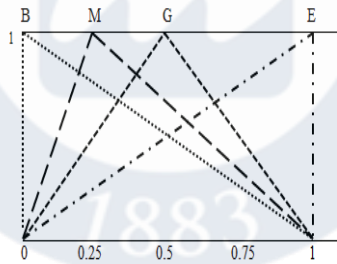


Figure 2. Triangular-shaped Membership Functions for 4 Linguistics Terms

By using fuzzy mode and fuzzy median transformation, the representative values of the four linguistic terms were calculated, and shown Table 4.

Table 4. Representative Values of Linguistic Terms

Transformation Method	Linguistic Terms			
	Bad (B)	Medium (M)	Good (G)	Excellent (E)
Fuzzy Mode	0	0.25	0.5	1
Fuzzy Median	0.293	0.387	0.5	0.5

Probabilistic Approach

First, mean and standard deviation calculated of each sample were calculated. The center line which is the grand mean and the mean of the sample standard deviations (MSD) were also calculated. The value of the constant A3, was taken 0.606 for sample size 50. Therefore, upper control limits (UCL) and lower control limits (LCL) were obtained. The results of these values were shown in Table 5.

Table 5. Fuzzy Probabilistic Approach Results

Transformation method		
Parameters	Fuzzy Mode	Fuzzy Median
CL	0.45	0.447
MSD	0.287	0.077
UCL	0.624	0.494
LCL	0.276	0.401
Width	0.348	0.093
A3	0.606	0.606

The fuzzy probabilistic control charts using fuzzy mode and fuzzy median can be seen on Figure 3.

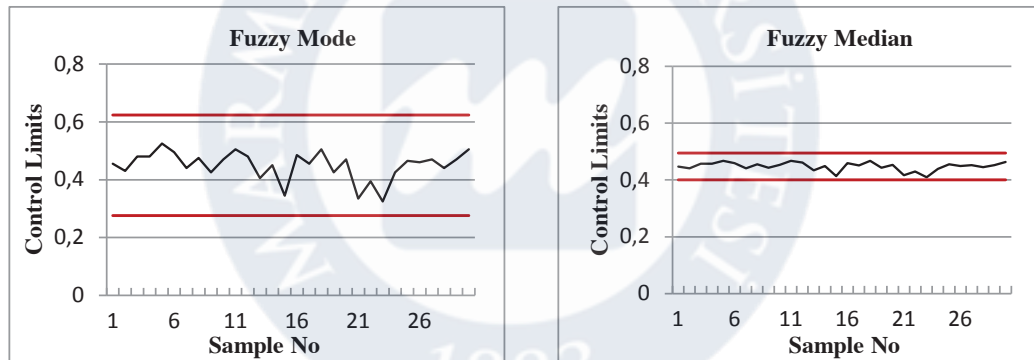


Figure 3. Fuzzy Mode and Fuzzy Median Probabilistic Control Charts

As seen on graphics, the process is in control for two transformation methods. Using fuzzy median and fuzzy mode for determining fuzzy probabilistic chart in our dataset does not give any different results.

Membership Approach

First, fuzzy membership control chart was obtained by applying membership approach to dataset. As seen on figure 4, the process is out of control at three occasions: Samples 15, 21 and 23. It can be clearly seen that the membership chart constructed using fuzzy mode as a transformation approach gives smaller interval between control limits than fuzzy probabilistic approach.

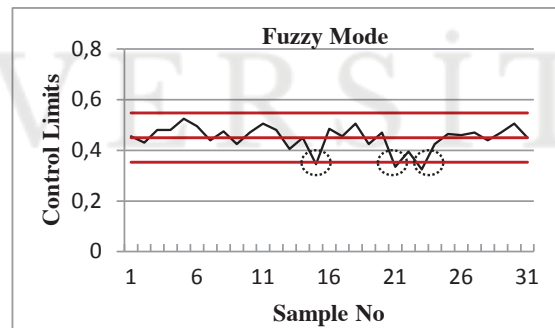


Figure 4. Fuzzy Mode Membership Control Chart

While calculating limits, the mean deviation was calculated approximately 0.338. Thus, CL, UCL, LCL and width were obtained as 0.45, 0.547, 0.353, and 0.194 respectively.



Conclusion

Control chart is a graphical tool which improves the quality level of the process by decreasing the variability and increasing the stability. It is an efficient tool to hold the process at control. Classical process control charts are appropriate if the data is exactly known and absolute. However, in some situations, we cannot have such certain data when human subjectivity plays a substantial role. Fuzzy set is inevitable in representing uncertainty, vagueness and human subjectivity. It is widely used if the data is expressed in linguistic terms such as “excellent”, “good”, “medium”, and “bad”.

In this study, quality characteristics of items packaging process were investigated using traditional statistical quality control and fuzzy logic. At first, generalized p chart, which is based on probability theory was applied. Then, fuzzy set theory in the study of control charts was applied to the same data to represent the uncertainty. Fuzzy set theory results are compared with the traditional approach to notice the difference in performance. The results of study showed that fuzzy control charts has better performance and more suitable than control chart approach. Because, it can detect abnormal status in the process than traditional quality chart technique. Therefore, by the aid of this theory, flexibility of the process was improved.

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