



A Heuristic Approach for Shelf Space Allocation Problem

A. Hande Erol Bingüler¹, Serol Bulkan², Mustafa Ağaoğlu³

Abstract

A shelf space allocation problem (SSAP) is a special form of multi-constraint knapsack problem. The main difference between knapsack problem and SSAP is that a knapsack problem has only capacity constraints, whereas an SSAP has some policy constraints in addition. Commercial space management systems use many different heuristic approaches for allocating shelf space due to NP-hard complexity of the SSAP. These heuristics are usually based on simple intuitive rules that could be easily used in practice to implement shelf space allocation decisions. In this paper, a new heuristic is developed to obtain good allocation of shelf space for different products in order to increase profitability under different constraints such as limited shelf space and elasticity factors.

Keywords: Evolutionary Algorithms, Heuristic Methods, Shelf Space Allocation Problem

Introduction

The knapsack problem is a combinatorial optimization problem. Given a set of items, each with a mass and a value, the goal is to determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible. Different knapsack problems exist in combinatorics, computer science, complexity theory, applied mathematics and optimization.

The knapsack problem has been studied since 1897. Tobias Dantzig was first referred this problem. Dantzig suggested the name that could have existed in myths before a mathematical problem had been fully defined. For all knapsack problems, efficient reduction algorithms have been proposed which enable one to fix several decision variables for objective functions [1]. There are many variations of knapsack problem like multi-objective knapsack problem, multidimensional knapsack problem, quadratic knapsack problem and subset-sum problem.

The multidimensional knapsack problem is similar to the bin packing problem. In this problem, a subset of items can be selected, however, in the bin packing problem, all items have to be packed to certain bins. The concept is that the items have multiple dimensions. This can be seen as a minor change, but it is not equivalent to adding to the capacity of the initial knapsack. This variation is used in many loading and scheduling problems in operations research and polynomial-time approximation scheme.

Despite the fact that the bin packing problem has an NP-hard computational complexity, optimal solutions to very large instances of the problem can be produced with hybrid algorithms. Many heuristics have been developed such as first fit algorithm, tabu search algorithm, genetic algorithm, etc. The goal of this problem, complementary to the minimum makespan scheduling problem, is to schedule jobs of various lengths on a fixed number of machines while minimizing the makespan, or equivalently to pack items of various sizes into a fixed number of bins while minimizing the largest bin size [2].

Shelf space allocation problem (SSAP) then also viewed as a two-dimensional packing problems which was studied by different researches. Gilmore and Gomory (1961) proposed the first model for two-dimensional packing problems, by modifying their column generation approach for one-dimensional packing problems [3]. Beasley (1985) also worked on bin packing problems and formulated an Integer

¹ A. Hande Erol Bingüler, Department of Industrial Engineering, Marmara University, Institute for Graduate Studies in Pure and Applied Sciences, Göztepe 34722, Istanbul, Turkey, hande.erol@marmara.edu.tr

² Serol Bulkan, Department of Industrial Engineering, Marmara University, Faculty of Engineering, Göztepe 34722, Istanbul, Turkey, sbulkan@marmara.edu.tr

³ Mustafa Ağaoğlu, Department of Computer Engineering, Marmara University, Faculty of Engineering, Göztepe 34722, Istanbul, Turkey, agaoglu@marmara.edu.tr



Linear Programming (ILP) model for two-dimensional packing problems [4]. Hadjiconstantinou and Christofides (1995) developed a similar model for this kind of problem [5]. Fekete and Schepers (1998) studied a new bin packing problem based on graph theory [6] and Lodi et.al. (2002) work on the special case of the problem where the products have to be packed by levels [7]. Michael and Moffitt (2013) show that item sizes and the capacity of bins span a vector of values, requiring that a feasible or optimal assignment of the items must satisfy capacity constraints in all dimensions [8].

In logistic field, shelf space is one of the most important resources to interest more consumers. Managing shelf space can not only reduce inventory level but also have stronger wholesaler relationship and higher customer satisfaction [9]. Shelf space, in which products are, is one of the major resources in retail environment [10]. So, the decision of shelf space management is an essential issue in retail operations management.

SSAP is a kind of multi-constraint knapsack problem. The main difference between knapsack problem and SSAP is that a knapsack problem has only capacity constraints whereas an SSAP has some policy constraints in addition. The knapsack model has applications for one dominant resource that manages budget or human resource planning, etc. Commercial space management systems use many different heuristic approaches for allocating shelf space due to NP-hard complexity of the SSAP. These heuristics are usually based on simple intuitive rules that could be easily used in practice to implement shelf space allocation decisions [11]. The concern for practicability and simplicity for these approaches, results in space allocation decisions that reach far from the optimum performance levels. According to the technological growth, the development of optimization approaches to solve SSAP has reached feasible solution to space management systems stage [12].

In retail store, SSAP is used as a decision problem to reach the possibly best objective using some operational constraints. The commercial space management systems use relatively simple heuristic rules to develop operating procedures designed easily to make decisions of shelf space allocation in practice [13]. Space allocation affects store profitability through both the demand function considering main and cross space elasticities together, and through the cost function (procurement, carrying and out-of-stock costs) [14]. Previous researches usually focus on a limited number of brands and only a few shelves [15].

Hwang et.al. (2009) integrated a mathematical model for the shelf space design and item allocation problem to maximize the retailer's profit. They used the shelf space design and allocation problem simultaneously considering location effect and space elasticity on demand. They developed heuristic solution procedures based on Genetic Algorithm [10].

In this study, a model is proposed as a comprehensive optimization model for allocating shelf space. This model used a modifying integer programming model for increasing its applicability in practice. The objective is to determine the best allocation of product items to the available shelf space to maximize objective function adding space elasticity factor. A Simulated Annealing (SA) algorithm is proposed to allocate items to shelf space, subject to given constraints.

Literature Survey

Carvalho (1999) studied arc flow formulation including side constraints for the one dimensional bin packing problem. A branch and price procedure that unifies deferred variable generation and branch and bound is used for the proposed model. OR Library test data sets are used for this research, a strong lower bound is derived and the linear relaxation leads to tractable branch and bound trees for these instances [16].

Lodi et.al. (1999) explored the class of problems arising from all combinations of the requirements that the items are obtained through a sequence of edge-to-edge cuts parallel to the edges of the bin. A heuristic algorithm and a combined tabu search approach adapted to a specific problem by changing the neighborhood [17].



Fekete and Schepers (2001) studied on dual feasible solutions and proposed a simple generic approach for obtaining fast lower bounds of bin packing problems. An asymptotic worst-case performance of 3/4 for a bound that can be computed in linear time for items sorted by size is proved. This study provides a general framework for establishing new bounds [18].

Retailers benefit from the optimum allocation of products into shelves in two ways; they reduce the costs of shelf replenishment and inventory, increase sales. The sales quantity of products depends on many factors such as location of the product within shelf, number of facings of product and adjacent products [15]. Anderson and Amato (1973) show that the companies increase the demand for a product by increasing the display area on the shelf. Table 1 shows research, algorithms and references.

Table 1. Research/Algorithms and references

Research/Algorithm	References
Demand model for a product depends on direct elasticity	Corstjens and Doyle (1981) [14]
Dynamic programming solution to a simplified version	Zufryden (1986) [11]
Greedy algorithm	Yang (2001) [12]
Squeaky Wheel Optimization algorithm	Lim et. al. (2004) [19]
Integrated mathematical model on multi-level shelves	Hwang et. al. (2005) [30]
Data mining approach and association rule mining	Chen and Lin (2007) [29]
Developed a model to two local supermarket chains using proprietary data	Fadıloğlu et. al. (2007) [31]
Study the effect of wholesalers pricing on allocation decisions of retailers	Martinez-de-Albeniz and Roels (2011) [32]
Model with elasticities at different aggregation levels	Eisend (2013) [28]

Yang (2001) presented a greedy algorithm to generate good solutions [12]. Lim et. al. (2004) improved Yang's heuristic approach and compared the original as well as the improved heuristics with three metaheuristic algorithms. Their algorithm that incorporates local search found best results [19].

Problem Definition

P_{ijk} is the profit of the product i in the right place of product j on shelf k , X_{ik} is the amount of product i on shelf k , then the objective can be formulated as:

$$\text{Max } P = \sum_{i=1}^I \sum_{j=1}^I \sum_{k=1}^K P_{ijk} * X_{ik} \quad (1)$$

Subject to

$$\sum_{i=1}^I a_i * X_{ik} \leq T_k \quad k = 1, 2, \dots, K \text{ (Shelf space constraint)} \quad (2)$$

$$L_i \leq \sum_{k=1}^K X_{ik} \leq U_i \quad i = 1, 2, \dots, I \text{ (Lower and upper bound for each product)} \quad (3)$$

$$\sum_{k=1}^K X_{ik} \in N \cup \{0\} \quad i = 1, 2, \dots, I \quad \forall \quad k = 1, 2, \dots, K$$



where;

$k = 1, 2, \dots, K$ the number of shelves

$i, j = 1, 2, \dots, I$ the number of products

T_k : the length of shelf k

a_i : the length of product i

L_i : the lower bound to allocate product i

U_i : the upper bound to allocate product i

Simulated Annealing

SA is one of the first available meta-heuristics. Therefore it is not astonishing that it is also the first one to be applied to QAP [20]. SA is a local search which relies on the process of statistical mechanics. Kirkpatrick et.al. (1983) are the first researchers who used the Metropolis algorithm as a heuristic to solve the traveling salesman problem. They proposed an iterative local search method, called SA, for solving combinatorial optimization problems [21].

The methodology between a many-particle physical system and a combinatorial optimization problem show similarities on two facts:

- The states of the physical system are represented by feasible solutions of the combinatorial optimization problem.
- The energy of the states of the physical system is represented by the objective function values.

Kirkpatrick et.al. (1983) proposed a method based on the experimental technique of the *annealing* used by the metallurgists to obtain a “well ordered” solid state, of minimal energy in order to solve NP-hard combinatorial optimization problems. This technique is based on the process of heating a material very fast and then reducing the temperature slowly. The SA method includes two parameters such as *annealing* and *temperature* coefficients [21][22].

The SA algorithm flow chart is shown schematically in the Figure 1. When this algorithm is adapted to the placement problem of components, simulated annealing operates a disorder-order transformation [22].

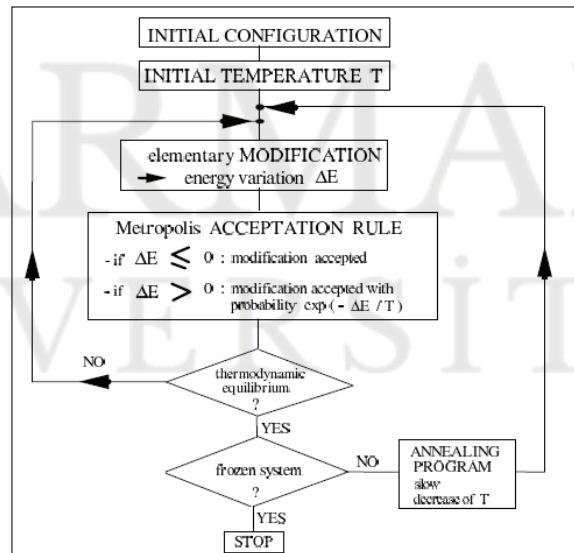


Figure 1. Flow chart of the simulated annealing algorithm [22]

In Figure 2, it is given a general algorithmic outline for SA.



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procedure Simulated Annealing
generate initial solution  $s$ ,  $s_{best} = s$ , initial value for  $T_0$ ,
 $n = 0$ ,
while outer-loop criterion not satisfied do
  while inner-loop criterion not satisfied do
     $s' = \text{GenerateRandomSolution}(s)$ 
     $s = \text{AcceptSolution}(T_n; s; s')$ 
    if ( $f(s) < f(s_{best})$ )
       $s_{best} = s$ 
    end
     $T_{n+1} = \text{UpdateTemp}(T_n)$ ,  $n = n + 1$ 
  end
return  $s_{best}$ 
end Simulated Annealing
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Figure 2. Outline of a simulated annealing algorithm [23]

SA starts from some initial solution s and generates in each step a new solution s' . This new solution s' is accepted or rejected according to an acceptance criterion. To implement a SA algorithm, some parameters and functions must be specified. Such as a random element of the neighborhood is returned by *GenerateRandomSolution* function and accepted by *AcceptSolution* function [23].

For an SA algorithm, an *annealing schedule* (often also called *cooling schedule*) is very important. In this schedule, T_0 is an initial temperature, new temperature obtains from previous temperature (*UpdateTemp*), the number of iterations must be performed at each temperature (inner loop criterion) and a termination condition (outer loop criterion) is used [23].

SA is of special appeal to mathematicians due to the fact that under certain conditions the convergence of the algorithm to an optimal solution can be proved. Mathematically, SA can be modeled using the theory of Markov chains. And SA algorithm converges asymptotically to the optimal solution [23].

Burkard and Rendl's motivated simulation procedure for combinatorial optimization problems is one of the first applications of SA to the QAP. It is shown that SA outperforms most of the existing heuristics for the QAP at that time. The corresponding algorithm yields a promising improvement of the trade-off between computation time and solution quality [24]. Thonemann and Bölte propose an improved SA algorithm for the QAP. A metaheuristic closely relates to SA, is also applied to QAP by Nissen and Paul [25].

Burkard and Rendl (1984) develop a general local search heuristic based on simulated cooling process applicable to any combinatorial optimization problem once a neighborhood structure is introduced in the set of feasible solutions [26]. In particular, Burkard et.al. (1998) apply SA to the QAP [27]. Other approaches for the SA apply to the QAP are Bos in 1993, Yip and Pao in 1994, Burkard and Çela in 1995, Peng et al. in 1996, Tian et al. in 1996 and 1999, Mavridou and Pardalos in 1997, Chiang and Chiang in 1998, Misevicius in 2000 and 2003, Tsuchiya et al. in 2001, Siu and Chang in 2002, and Baykasoğlu in 2004 [25]. These studies differ from each other on implementation of cooling process or the thermal equilibrium.

Advantages of SA method are the flexibility on the evaluations of the problem and the easiness of implementation. On the other hand, the main disadvantages of SA are the difficulty of adjustments of temperature decrease. SA obtained excellent results of big size problems [22].



Simulated Annealing Application

The initial temperature is defined in such a way that the proportion of, the accepted cases to the whole studied cases (γ), in Markov chain has the value of 0.2-0.5 according to different problems. The number of iterations for a specific temperature is proportional to the number of acceptances in an inner loop instead of the number of tries for generation and evaluation. The number of acceptances required for search in the inner loop of the algorithm decreases according to the temperature reduction.

The freezing temperature or convergence condition definition is very important in increasing the speed and accuracy in the search process. If the stop condition of the algorithm is not defined effectively, the algorithm will be stopped sooner which as a result reduces the accuracy or the convergence of the algorithm will be announced by delay which results in the speed reduction. The way of determining the convergence conditions for algorithm is explained in different methods and various criteria are discussed in the literature. The stop condition is defined when in two sequential searches in Markov chain, there is no change in the best obtained (total cost) result.

A kind of a greedy algorithm is used for local search at the end of the SA algorithm. This algorithm gives the final response of the algorithm. For the determination of the negative elements' displacement priority, the *lowest negative element* is selected greedily. The proposed algorithm starts with the initial temperature proportional to 20% of acceptances to the whole situations, ($\gamma = 0.2$) for categories (A) and (B), proportional to 50% of acceptances to the whole situations, ($\gamma = 0.5$) for category (C) and proportional to 40% of acceptances to the whole situations, ($\gamma = 0.4$) for category (D).

The performed operation in the existing loop in defining the initial temperature, is exactly the same operation used in the search engine. This means that it starts with a random variable and after applying the switching operator, the acceptance terms of the algorithm will be checked. In the case of acceptance, the previous generation will be replaced by a new one and the operation will be continued until the Markov chain ends and at the end, it is the ratio of accepted, to the total states that represents the desired ratio. If this ratio is sufficient, the loop will stop, but otherwise, a new ratio will be calculated for the increased temperature by the stepped increase of the temperature and repetition of the above stages. This temperature increase will be continued until the ratio of the accepted states to the total states in a single Markov chain, reaches the ratio defined at the beginning of the algorithm.

In the first inner loop, the iterations is finished when a specific number of acceptances occur related to $A_0 = 2000n/(1-\gamma)$. In each temperature reduction the number of acceptance reduces with the equation $A_k = 0.8 (5 * A_0)^{1/k}$. More tries are performed in effective temperatures in the search process. The repetition rate reduction is resulted by trial and error.

In the inner loop for producing the new generation, the Switching operator is used. It seems that this operator can find the best possible result. Optimization of process is performed in Java program.

Results and Conclusion

Simulating Annealing algorithm is used for 100 times to evaluate their efficiency and the findings are summarized on Table 2.

Table 2. Comparison of algorithm results

Problem name	Min z	Mean z	Max z	Average run time
Simulated Annealing Algorithm	106.85	162.38	193.85	200.18
Genetic Algorithm (Bilsel, et. al., 2013) [33]	145.95	162.38	176.25	253.13
Greedy (Yang, 2001) [12]	135.90	135.90	135.90	
Greedy with improvements (Ayhan et al., 2007) [34]	146.10	146.10	146.10	



As can be seen from Table 2, the proposed Simulated Annealing algorithm performs better than Yang's and Ayhan et.al.'s heuristics. Its average performance is the same as Bilsel et.al.'s genetic algorithm. However, for the best (max) solution, SA outperforms Bilsel et. al.'s genetic algorithm.

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